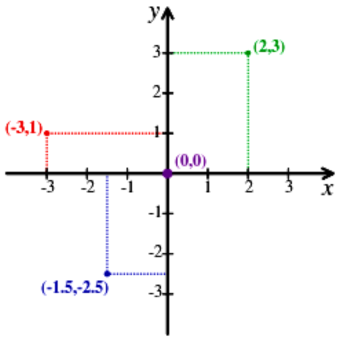
6.9 THE CARTESIAN COORDINATE SYSTEM

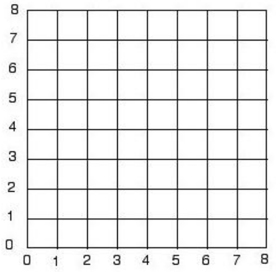
Slope can also be calculated from the values of points on a line using a formula. How is this done? First, you must know the coordinates of 2 points on that line, and the use the formula. Before investigating the formula, review of the Cartesian coordinate system is necessary.

The **Cartesian coordinate system** is a grid that identifies the location of a point by a pair of numbers which are a specific distance from a fixed point called the **origin** (For our purposes, we are only using 2-dimensions but 3-dimensions can be used!).



The grid is shown, with several points marked. Notice that the points are located by 2 lines, one horizontal and the other vertical. When marking a point on a grid system, the first number indicates how far right or left of the origin (0,0) the point is (x-coordinate), and the second number indicates how far up or down the point is (y-coordinate). With these two values, any point can be plotted.

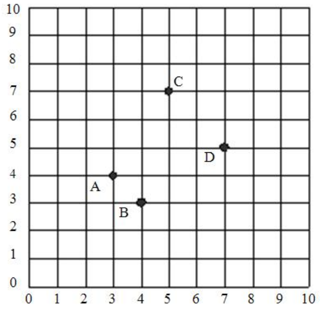
Each reference line is called an axis, and they cross at the origin. For each axis, one half is positive (right and up) and the other half is negative (left or down). This is like two number lines – one horizontal and one vertical. You can see this reflected in the points plotted to the right.



For the purposes of this course, we will only deal with the **FIRST** quadrant of the grid, where both numbers have a positive value.

Identifying the locations of points on this grid is called naming the points. As mentioned before, we put the coordinates in parentheses, with the x-coordinate first and the y-coordinate second.

Example 1: Name the coordinates on the grid.

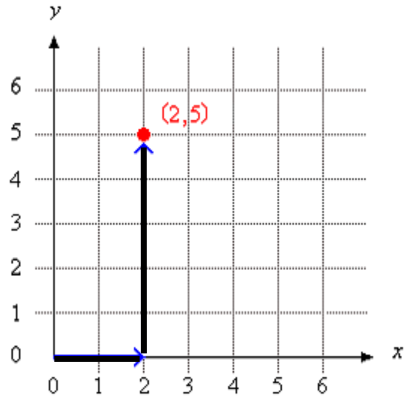


Solution: First, count from the origin to the right, and second count how far up each point is.

A (3, 4)
B (4, 3)
C (5, 7)
D (7, 5)

Example 2: Plot the point (2, 5).

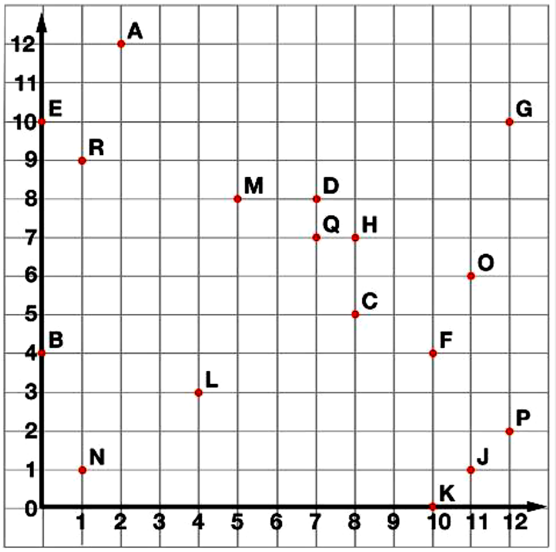
Solution: Start at the origin (0, 0) and move 2 units to the right and then 5 units up. Mark the location with a small dot as shown.



THIS IS REALLY IMPORTANT TO REMEMBER!

ASSIGNMENT 6.9 – ORDERED PAIRS

Using the grid below, answer the following questions.



1) Name the letter of the point located at each of the following coordinate pairs.

______ (10, 0) ______ (5, 8)

______ (8, 7) ______ (2, 12)

______ (7, 7) ______ (0, 10)

2) Write the ordered pair for each given point.

N __________ R __________

O __________ G __________

J __________ F __________

3) Plot the following points on the coordinate grid.

S (3, 6) T (8, 2) U (10, 8)

W (5, 5) X (4, 2) Y (2, 7)

6.10 REVIEW – SIMPLIFYING FRACTIONS

To simplify a fraction, divide the numerator and denominator by a common factor. Easy common factors to start with are 2 for even numbers, 3, or 5. If the resulting fraction cannot be divided by any other common factor, then it is in **lowest terms**. If it can be divided again by another common factor, keep repeating the process until it is in lowest terms.

Example 1: Simplify $\begin{array}{l} \underline{18} \longleftarrow \text{numerator} \\ 27 \longleftarrow \text{denominator} \end{array}$

Solution A: $\begin{array}{l} \underline{18} \div 9 = \underline{2} \\ 27 \div 9 = 3 \end{array}$ Simplify, using a factor of 9

Solution B: $\begin{array}{l} \underline{18} \div 3 = \underline{6} \div 3 = \underline{2} \\ 27 \div 3 = 9 \div 3 = 3 \end{array}$ Simplify, using a factor of 3, twice

ASSIGNMENT 5.10 – SIMPLIFYING FRACTIONS

1) Simplify these fractions to their lowest terms. Show your work!

a) $\frac{4}{16}$

b) $\frac{3}{12}$

c) $\frac{25}{75}$

d) $\frac{15}{21}$

e) $\frac{8}{18}$

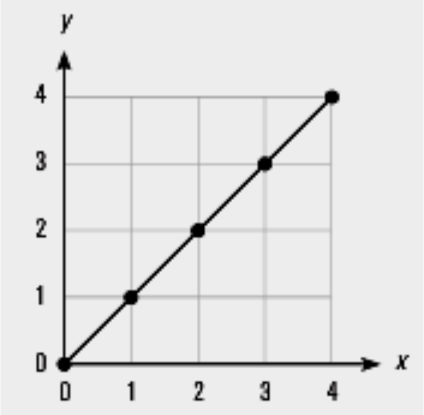
f) $\frac{45}{100}$

g) $\frac{20}{50}$

h) $\frac{3}{21}$

i) $\frac{7}{56}$

6.11 CALCULATION OF SLOPE OF A LINE

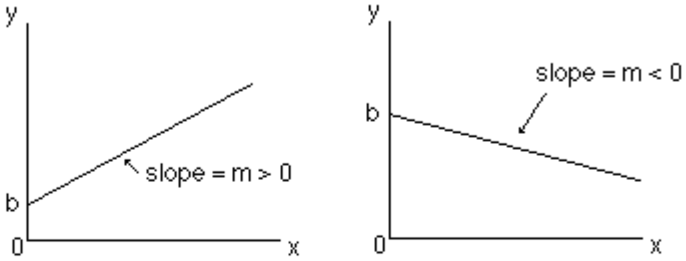


When calculating slope, you will always be working with a straight line. These lines will have identifiable points plotted along them.

To calculate the slope of such a line, two points are needed. You can see from this example that there are 5 usable points. Any 2 can be used – it doesn't matter which ones. And it doesn't matter which point you start with. Usually it is a good idea to choose the point with the bigger values.

When you choose the two points, they must be in the form (x , y) – that means that you read the x-coordinate first followed by the y-coordinate.

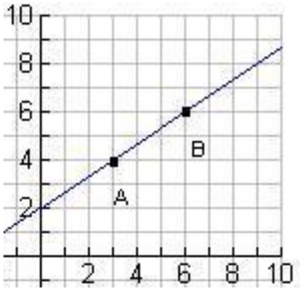
Slopes can be <u>positive</u> or <u>negative</u>. If the line goes up to the right, the slope is positive (graph on the left). If it goes down to the right, the slope is negative (graph on the right). Slope of a line has no units, it is just a numerical value; just a number.



The slope is calculated as the change in the vertical distance divided by the change in the horizontal distance. The letter "*m*" is used to represent slope. The formula used to calculate slope is:

$$\boxed{m = \frac{y-y}{x-x}}$$

<u>Example 1</u>: Using the formula to calculate slope, find the slope of the line shown on the graph.



<u>Solution</u>: To calculate the slope of a line, choose 2 points on that line. It is easier and more accurate to choose points that lie on the intersection of the two grid lines. The two points are marked on the graph.

Point A (3, 4) Point B (6, 6)

The slope is the change in the values as we move from point A to point B. The symbol Δ*x* ("delta *x*") means how much the *x*-coordinate will change (as we move from A to B). And the symbol Δ*y* ("delta y") means how much the *y*-coordinate will change.

The slope of the line, *m*, is:

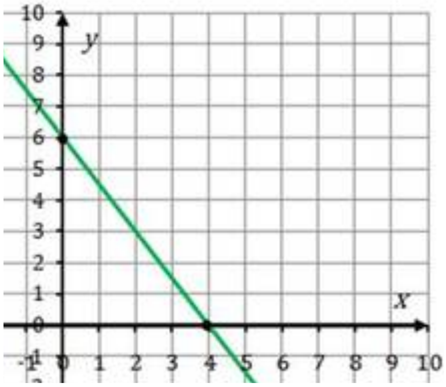
$$m = \frac{\Delta y}{\Delta x} = \frac{y-y}{x-x} = \frac{\text{Change in the } \textbf{rise}}{\text{Change in the } \textbf{run}}$$

Remember, the rise is always the change in the y and the run is always the change in the x. It does not matter which point you start with.

So, using the coordinates above, **B** is (6, 6), and **A** is (3, 4), then the slope of that line is

$$m = \frac{\Delta y}{\Delta x} = \frac{y-y}{x-x} = \frac{6-4}{6-3} = \frac{2}{3} = 0.\overline{6} = 0.7$$

<u>Example 2</u>: Calculate the slope of the following line.



<u>Solution</u>: Choose 2 points and use the slope formula to calculate the answer.

There are several points that can be used: two are marked on the grid (4, 0) and (0, 6). Another point that could be used is (2, 3). All the other potential points only cross one grid line so the other value would be estimated. This is not a good choice.

Use (4, 0) and (2, 3).

$$m = \frac{\Delta y}{\Delta x} = \frac{y-y}{x-x} = \frac{3-0}{2-4} = \frac{3}{-2} = 3 \div -2 = -1.5$$

Because the line slopes down to the right, it has a negative slope.
The slope of this line is –1.5.

<u>**NOTE**</u>: If we had chosen any combination of the three points, (4, 0), (0, 6), or (2, 3) the answer would have been the same. Here is proof.

Use (0, 6) and (2, 3).

$$m = \frac{\Delta y}{\Delta x} = \frac{y-y}{x-x} = \frac{3-6}{2-0} = \frac{-3}{2} = -3 \div 2 = -1.5$$

If you are given the slope of a line and the coordinates of any point on the line, it is possible to plot that line.

Example 3: Plot a line on the graph that goes through (1, 3) and has a slope of 2. Write the coordinates of 2 other points that are on that line.

Solution: First plot the point on the grid.

Next, use the slope to plot other points as follows:

Make the slope into a fraction by using a denominator if 1.

Remembering that $\text{slope} = \frac{\text{rise}}{\text{run}}$, fill this in with your slope.

So, slope $= \frac{\text{rise}}{\text{run}} = \frac{2}{1}$

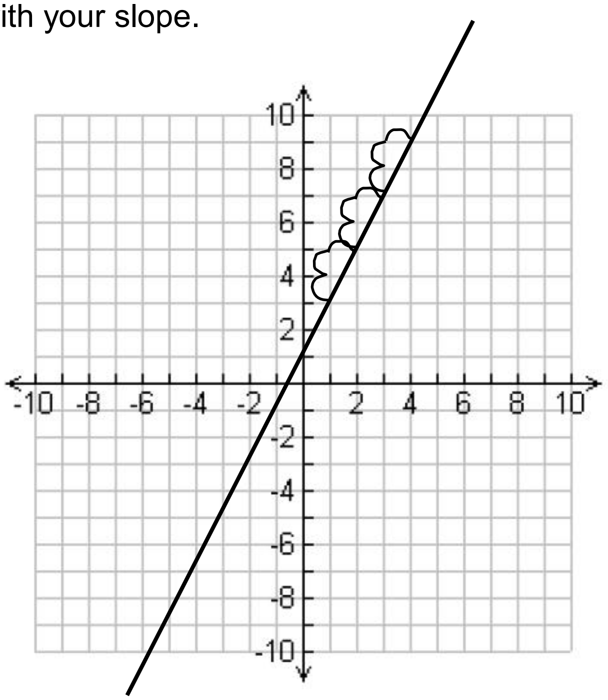
For this line then, the rise = 2 and the run = 1. Use this to plot new points on the graph by starting at the original point (1, 3) and RISING 2 and RUNNING 1 from each point to the next new one. A minimum of 3 points is needed.

Join the points with a straight line covering the whole grid space.

Write the coordinates of any 2 points that fall on the line. Examples include:

(2, 5) or (4, 9) or (0, 1) or (-1, -3)

Remember to choose points where the line is on top of a cross of both gridlines.



ASSIGNMENT 6.11 – CALCULATING SLOPE OF A LINE

1) Calculate the slope for each of the following pairs of points. State whether the line would slope up or down to the right.

a) A (2, 2) B (6, 3)

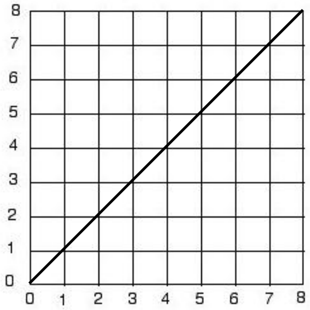
b) C (5, 1) D (3, 2)

c) E (12, 8) F (2, 10)

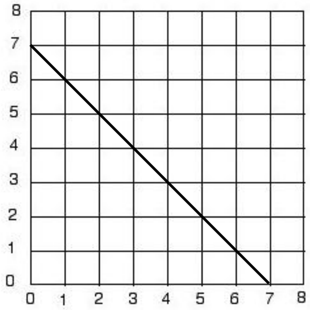
d) G (1, 4) H (3, 1)

2) For each of the following graphs, state whether the slope is positive or negative. Then calculate the slope.

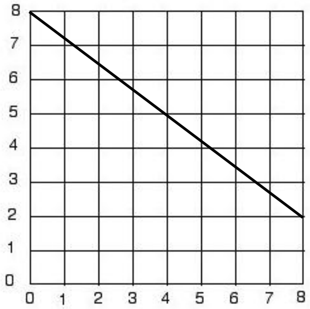
a)



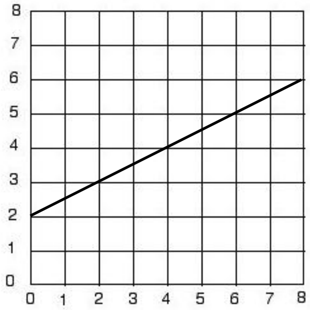
b)



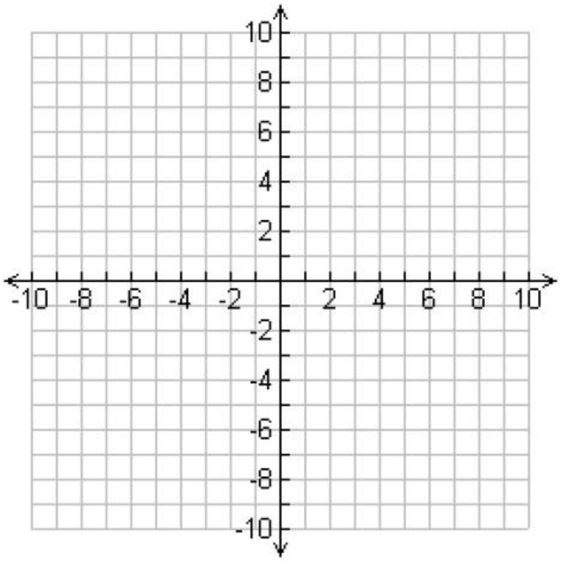
c)



d)



3) Plot a line on the graph that goes through (4, 2) and has a slope of 3. Write the coordinates of 2 other points that are on that line.


10
8
6
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